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MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. XXII. *Jan.* 10, 1862. No. 3.

Dr. LEE, President, in the Chair.

The Rev. Thomas Pyne, Hook, near Kingston-on-Thames;
James McDowell, Esq., Perse Grammar School, Cambridge;
William Wray, Esq., 25 Torriano Terrace, Kentish Town;
C. H. Lake, Esq., Oxford House, King's Road, Chelsea;
Signor Julian Vertu, Lincoln;
John Kidd, Esq., Sherborne, Dorset; and
James Breen, Esq.,

were balloted for and duly elected Fellows of the Society.

An Account of Experiments on Solar Radiation.—Appendix describing the Method employed to discover the Influence of the Air in the Cooling of the Sun Thermometer X., and of Ascertaining the Correction required to be Applied to Observations of r, so as to reduce them to a Vacuum. By John James Waterston, Esq.

Fig. 2[*] is an upright section of the little apparatus or vacuum-bath. It consists of a cylindrical vessel of brass

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[*] See last number of the *Notices*: the scale is 1 in. = 4 in.—ED.

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coated internally with lamp-black. The lid was ground air-tight to the upper edge, and had a stuffing-box in the centre, through which the stem of the thermometer was passed. A stop-cock enabled a communication to be made with an air-pump. With a plentiful supply of lard to the stuffing-box and ground surfaces, a good vacuum could be maintained for a day unimpaired.

The time was measured by the beats of a clock: to register the number of these at each degree as the mercury of the thermometer descended, a scale of equal parts was prepared extending to 1000, and with distinguishing marks at each 5, 10, 50, and 100. Then with a pencil in the right hand over the scale and a magnifying glass in the left over the scale of the thermometer, I counted the beats, and when the mercury came to the line of a degree, made a mark on the scale of equal parts opposite the number of beats, and at the same time continued to count on; *e. g.* if 57 was the number when the mercury came to a line, a pencil-mark was made at 57 on the scale of equal parts, and the counting went on,—58, 59, &c. until the mercury came to the next line.

Thus, not a beat was lost from beginning to end, and the accuracy was only limited by the accuracy of the divisions on the scale of the thermometer. Indeed, this method is a severe test to the equality of the divisions, because the reciprocal of the differences in the number of the beats for each degree, if laid off as ordinates to the total number of beats, ought to range in a straight line, and any saw-like irregularities indicate inaccuracy in the divisions of the scale of the thermometer. To heat the bulb of the thermometer a funnel was placed over the small flame of a Bunsen; then holding the plate (having the thermometer fixed in its place) by means of the stop-cock, the bulb was brought over the top of the funnel until the mercury had risen to near the top of the scale. The plate was then quickly placed on the cylinder, communication made with the air-pump, and the air exhausted from the cylinder by 20 strokes, the capacity of the pump being about one-third that of the cylindrical vessel or vacuum-bath. The vessel thus exhausted was placed in a water-bath, the temperature of which was ascertained at the beginning and end; the difference seldom amounting to $\frac{1}{10}$th of a degree.

The following table exhibits two series of observations on the cooling of the sun thermometer, x, in the vacuum, and in air taken while the water-bath remained steady at 48°. This basal temperature being at an exact degree enables the rate of cooling to be studied easily without fractional parts or interpolation:—

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Temp. X.	Beats in Vacuum.	r.	Beats in Air.	Temp. X.	Beats in Vacuum.	r.	Beats in Air.
90°		42°	0	90°		42°	
85°	0	37°	36	65°	323	17°	270
4	12	36		4	348	16	291
3	24	35		3	377	15	312
2	36	34		2	406	14	336
1	48	33		1	437	13	360
80	60½	32	78	60	470	12	385
9	73	31		9	510	11	414
8	86	30		8	549	10	448
7	100½	29		7	594	9	484
6	115	28		6	643	8	527
75	130½	27	129	55	704	7	576
4	145½	26		4	765	6	630
3	161	25		3	843	5	695
2	179½	24		2	...	4	780
1	197	23		1	...	3	
70	214	22	191	50	...	2	
9	235	21		9	...	1	
8	255	20		48	...	0	
7	276	19					
6	298	18					

Thus, 53, being the last observation of the vacuum cooling, corresponds with $r = 5°$; then

At $r = 5$ we have 843 beats
$r = 10$,, 549 ,,
$r = 20$,, 255 ,,
$r = 40$,, -32 ,,

Difference 294
,, 294
,, 287

Mean Difference 291·7

Again, beginning with $r = 7°$,

At $r = 7$ we have 704 beats
$r = 14$,, 406 ,,
$r = 28$,, 115 ,,

Difference 298
,, 291

Mean Difference 294½

And beginning with $r = 9$,

At $r = 9$ we have 594 beats
$r = 18$,, 298 ,,
$r = 36$,, 12 ,,

Difference 296
,, 286

Mean Difference 291

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The numbers in the column of differences ought to be the same if the law of cooling in a vacuum is perfectly true, if the vacuum is complete, and if the graduation of the thermometer is correct. The difference between them is so small that the result must, I think, be deemed satisfactory.

Let us study the same differences with the cooling in air:—

At $r = 5^\circ$	we have 695 beats	1st Difference 247	2d Difference 19
$r = 10$	,, 448 ,,	,, 228	,, 22
$r = 20$	,, 220 ,,	,, 206	
$r = 40$	,, 14 ,,		

$r = 7^\circ$	we have 576 beats	1st Difference 240	2d Difference 23
$r = 14$	,, 336 ,,	,, 217	
$r = 28$	,, 119 ,,		

$r = 9^\circ$	we have 484 beats	1st Difference 230	2d Difference 20
$r = 18$	,, 254 ,,	,, 210	
$r = 36$	,, 44 ,,		

Thus, it appears that in air the cooling takes place in a ratio greater than r, the 1st difference of the times diminishing and the 2d difference slightly increasing between 247 and 210. The limiting value of the 1st difference must be 294 when $r = 0$, and 294 minus the 1st difference increases nearly as $\sqrt{r}$. An empirical formula constructed in conformity with this ratio cannot differ much from the observations.

Let Δ represent 1st difference and g a constant,

$$\Delta = 294 - g\sqrt{r} \quad \text{and} \quad g = \frac{294 - 247}{\sqrt{7\frac{1}{2}}} = 17\cdot162 = \frac{294 - 210}{\sqrt{27}} = \&c.\ \text{(nearly)}.$$

Δ represents the logarithm of 2, so that c being a constant, we have $\log r = \frac{\Delta}{c}$. In the curve that represents the cooling in air we may assume a small arc of it to coincide with a true logarithmic curve or the curve of cooling in a vacuum, and we have to find the value of r, the ordinate of the true logarithmic curve at the given point.

The logarithmic curve is defined by the equation

$$c \log \frac{r_1}{r_0} = t_1 - t_0 = \frac{\Delta}{\log 2} \log \frac{r_1}{r_0}.$$

Let

$$r_0 = (r_1 - 0^\circ\cdot 1).$$

To find the value of r_v corresponding to a given r_a (r_v vacuum, r_a air), we require to compute the value of $t_1 - t_0$, employing $\Delta_a = 294 - g\sqrt{r}$ in the equation

$$\frac{\Delta_a}{\log 2} \log \frac{r_a}{(r_a - 0^\circ\cdot 1)} = t_1 - t_0;$$

then, with this value of $t_1 - t_0$, and with $\Delta_v = 294$, find the value of r_v in the equation

$$t_1 - t_0 = \frac{\Delta_v}{\log 2} \cdot \log \frac{r_v}{(r_v - 0^\circ\cdot 1)}.$$

The direct equation is

$$\Delta_a \left\{\log r_a - \log (r_a - 0^\circ\cdot 1)\right\} = \Delta_v \left\{\log r_v - \log (r_v - 0^\circ\cdot 1)\right\}$$

and

$$\frac{\Delta_a}{\Delta_v} = 1 - \frac{g}{294}\sqrt{r_a} = 1 - \frac{\sqrt{r_a}}{17\cdot 131}.$$

Hence r_v may be ascertained by inspecting the differences of a table of logarithms; and it was from these that a scale was constructed for reducing the values of r taken in air to what they would be if taken in a vacuum, where the emission of heat was by radiation alone.

The cooling of the sun thermometer in air when fixed in its place in the tube, as in fig. 1, was found to be exactly the same as when fixed in the cylinder, fig. 2, unexhausted.

A chemical thermometer with cylindrical reservoir was tried in the vacuum-bath, and the cooling was found to take place exactly in the logarithmic curve. It is difficult to adjust the vacuum-bath in time to observe a high value of r, but good observations were obtained from $r = 190^\circ$ downwards; so there is little doubt that the law of cooling by radiation is general and independent of the shape of the cooling body. I purpose extending these observations with different surfaces. One result is interesting, as showing the perfect reciprocity of the radiation, viz. a gilt bulb radiating against a blackened metallic surface loses heat at the same rate as a blackened bulb against a bright metallic surface; the rate being slower than when both are blackened.

Another fact that it is well to keep in view is that, although the uncoated glass bulb radiates as quickly as the same coated with black, it does not absorb the incident rays of heat to the same degree; nearly one half being reflected without entering the glass.

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Observations on Solar Radiation.

	Apparent Time. Latitude, 55° 58′ N. Longitude, 0h 12m 44s W.	Observed Values of r.	Ditto corrected for Talc Film.	Correction to Reduce to a Vacuum.	Value of r Reduced to a Vacuum.	Reciprocal of Ditto.	Apparent Altitude of Sun's Centre.	Cosecant of Apparent Zenith Distance.	Remarks
		r_o	r_a	p	r_v	$\frac{1}{r_v}$	App. Alt.		
1861.	h m	°	°	°	°		° ′		
July 28	4 25 P.M.	17·5	20·6	+7·4	28·0	·0357	28 49	2·075	Bulb coated with lamp-black and turpentine; surface moist.
	4 36	16·5	19·5	6·8	26·3	·0380	27 16	2·183	
	4 56	15·5	18·3	6·2	24·5	·0408	24 30	2·411	
	5 26	14·0	16·5	5·2	21·7	·0461	20 19	2·880	
	6 18	13·2	15·5	4·7	20·2	·0495	13 11	4·385	
	7 26	7·2	8·5	1·7	10·2	·0980	4 27	12·888	
30	6 36 P.M.	11·8	13·9	+3·9	17·8	·0562	10 41	5·394	
	6 45	10·8	12·7	3·3	16·0	·0625	9 12	6·255	
	7 13	6·6	7·8	1·4	9·2	·1087	5 44	10·010	
31	Noon	23·0	27·2	+11·4	38·6	·0259	52 20	1·263	Extra favourable conditions.
	5 41 P.M.	15·0	17·7	5·8	23·5	·0425	17 38	3·301	Ditto.
	6 6	14·0	16·5	5·2	21·7	·0461	14 3	4·119	
	6 29	13·0	15·3	4·6	19·9	·0502	11 6	5·194	
	7 18	9·0	10·6	2·5	13·1	·0763	4 36	12·47	
	7 34	6·0?	7·1	1·3	8·4	·1190	2 52		From July 31, noon, to Aug. 5, noon, the bulb coated with China ink and gum solution; surface dry.
Aug. 1	9 51 A.M.	20·5+?	24·2	+9·5	33·7	·0297	44 58	1·415	
4	11 1 A.M.	20·0	23·6	9·1	32·7	·0306	49 35	1·313	
	6 36 P.M.	11·3	13·4	3·6	17·0	·0588	9 15	6·221	
5	Noon	20·2	23·8	9·2	33·0	·0303	51 0	1·287	Extra favourable.
B	3 44 P.M.	11·3	13·3	3·6	16·9	·0592	33 14	1·825	Bulb uncoated.
B	4 30	9·7	11·4	2·7	14·1	·0709	27 10	2·190	Ditto.

All the above, taken with thermometer Y in the sun, and its correction for air, has been assumed to be the same as that experimentally determined for thermometer X in the subsequent observations.

In the following observations the thermometer in sun was X, with bulb coated with China ink:—

		r_o	r_a	p	r_v	$\frac{1}{r_v}$	App. Alt.	cosec.	
1861.	h m	°	°	°	°		° ′		
Aug. 9	3 34 P.M.	14·4	17·0	+5·5	22·5	·0444	33 7	1·830	
10	8 50 A.M.	15·2	17·9	+5·9	23·8	·0420	35 56	1·704	Taken from a consecutive series of 18 observations.
	9 8	15·5	18·3	6·1	24·4	·0410	38 5	1·621	
	9 23	16·2	19·1	6·6	25·7	·0389	39 48	1·562	

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1861.	h m	r_o	r_a	p	r_v	$\frac{1}{r_v}$	App. Alt.	cosec.	
		°	°	°	°		° ′		
13	4 56 P.M.	12·8	15·1	+4·5	19·6	·0510	21 1	2·787	Not less, perhaps greater value of *r*.
	6 56	4·8	5·7	0·9	6·6	·1515	4 37	12·42	
	7 27	2·2	2·6	0·3	2·9	·3448	0 53		Sunset.
15	6 1 P.M.	9·5	11·2	2·7	13·9	·0719	11 25	5·05	
	7 4	3·8	4·5	0·6	5·1	·1961	3 4	18·69	
16	5 16 A.M.	4·4	5·2	0·8	6·0	·1667	5 29	10·465	Sun rises perfectly clear of clouds.
	9 39	14·6	17·2	5·6	22·8	·0439	39 51	1·561	
	11 3	15·0	17·7	5·8	23·5	·0425	46 20	1·382	
21	7 20 A.M.	9·8	11·6	2·9	14·5	·0690	19 0	3·071	Consecutive series graphically projected and equalised.
	7 40	10·7	12·6	3·3	15·9	·0629	21 45	2·698	
	8 0	11·6	13·7	3·8	17·5	·0571	24 29	2·413	
	8 20	12·3	14·5	4·2	18·7	·0535	27 9	2·191	
	8 40	13·0	15·3	4·6	19·9	·0502	29 45	2·015	
	9 0	13·5	15·9	4·9	20·8	·0481	32 14	1·874	
	9 20	13·8	16·2	5·0	21·2	·0472	34 37	1·760	
	9 40	14·1	16·6	5·3	21·9	·0457	36 50	1·668	
	h m s								
20	6 21 16 P.M.	6·6	7·7	+1·4	9·1	·1099	7 21	7·817	Consecutive series not equalised.
	6 23 16	6·5	7·6	1·4	9·0	·1111	7 4	8·128	
	6 24 31	6·4	7·5	1·4	8·9	·1124	6 55	8·304	
	6 26 16	6·1	7·2	1·3	8·5	·1177	6 41	8·592	
	6 27 16	6·0	7·1	1·2	8·3	·1205	6 33	8·767	
	6 28 36	6·1	7·2	1·3	8·5	·1177	6 22	9·018	
	6 31 0	5·7	6·7	1·1	7·8	·1282	6 2	9·514	
	6 37 16	4·8	5·7	0·8	6·5	·1538	5 12	11·034	
	6 39 0	4·7	5·6	0·8	6·4	·1562	4 59	11·512	
	Small cloud passes over the sun.								
	6 43 56	3·4	4·0	0·5	4·5	·2207	4 22	13·134	
	6 45 46	3·3	3·9	0·5	4·4	·227	4 5		
	6 47 16	3·2	3·8	0·5	4·2	·235	3 54		
	6 49 16	3·0	3·6	0·5	4·0	·249	3 38		
	6 51 0	2·8	3·3	0·4	3·7	·272	3 25		
	6 52 16	2·7	3·1	0·3	3·5	·287	3 15		
	6 54 0	2·4	2·9	0·3	3·2	·312	3 2		
	6 55 45	2·2	2·6	0·3	2·9	·348	2 48		
	6 58 46	1·9	2·2	0·2	2·4	·408	2 30		
	7 1 16	1·6	1·9	0·2	2·1	·483	2 7		
	7 4 0	1·1	1·3	0·1	1·4	·714	1 48		
28	Noon	12·5	14·8	4·3	19·1	·0524	43 42	1·447	
	0 44 P.M.	13·2	...	...	...	...	...	...	*r* sensibly augments after culmination.

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In the following, thermometer Y was in sun, with its bulb coated with lamp-black in gold size:—

		r_o	r_a	p	r_v	$\frac{1}{r_v}$	App. Alt.	cosec.
1861.	h m	°	°	°	°		° '	
Aug. 6	8 7 A.M.	16·2	19·1	6·6	25·7	·0389	31 17	1·926
	9 8	18·0	21·2	7·7	28·9	·0346	42 21	1·484
	9 32	18·3	21·6	8·0	29·6	·0338	45 12	1·409

Note.—Referring to the method of computing the Sun's potential temperature, described in the proceedings of the Society for March 1860, and employing the same rule with R, the extra atmospheric value of r equal to 70° at Earth's mean distance, we arrive at 12,880,000° as the potential temperature of its radiating surface.*

If we expose the flame of a bat's-wing-jet to one ball of a differential thermometer, the effect is the same, whether the broad side or the narrow side of the flame is presented, as I have found on trial. Now the potential temperature being equal to the product of r by the reciprocal of the angular space occupied by the flame, it is in the one case about five times greater than in the other. In the same way we might compute the potential temperature of an angular space occupied by many thousand flames placed one behind the other, extending in a line from the observer, and probably we should find it cumulative in the ratio of the number of flames.

From observations I have made on gas flames with the radiation meter, fig. 1, it would seem to require about 4000 bat's-wing-flames ranged behind each other to give a potential equal to that of the Sun.

If the upper radiating matter of the Sun is in any degree transparent or permeable to radiation from lower strata, it is obvious that the *actual* temperature may thus be much below the *potential*.

26 *Royal Crescent, Edinburgh, November* 25, 1861.

Note on a Theorem of Jacobi's, in relation to the Problem of Three Bodies. By A. Cayley, Esq.

The following theorem of Jacobi's (*Comptes Rendus*, t. iii., p. 61 (1836)) has not, I think, found its way in an explicit form into any treatise of physical astronomy. The theorem is as follows, viz. "Consider the movement of a point without mass round the Sun, disturbed by a planet the orbit of which

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* There is a typographical error in the *Monthly Notices*, page 201, line 32; for $x = 918000$, read $x = 9{,}180{,}000$.